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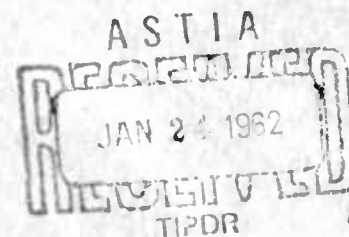
ON THE EFFECTIVENESS OF SEARCH  
ALGORITHMS BASED ON SAMPLES  
OF CONTROLLED DURATION  
(Sequential Detection)

Translated from the Russian by L. E. Brennan

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TRANSLATOR'S PREFACE

This article, written by U. B. Kobzarev and A. E. Basharinov, is translated from the September 1961 issue of Radiotekhnika i Elektronika (Radio Engineering and Electronics), Vol. VI, No. 9, pp. 1411-1419, published by the USSR Academy of Sciences.

Sequential detection promises to be a useful technique in increasing the search range of phased-array radars, which are currently under development. RAND workers have done some analytical work in this field, and the present translation was undertaken in support of that work.

The translation should be of interest to workers at Rome Air Development Center. Copies will also be deposited with the Office of Technical Services, U.S. Department of Commerce.

### RÉSUMÉ

Search procedures are considered which employ trial steps (samples) of controlled duration.

Indices are defined which characterize the effectiveness of these controlled search procedures.

The influence of the capacity (i.e., variety of forms of signal) on these indices is estimated.

1. In automatic measuring systems, tracking and self-tuning systems, as well as in systems for automatic radar detection, the search procedure plays an important role.

Simultaneously testing for all possible responses (e.g., tuning frequencies), as a rule leads to an excessively cumbersome system. As a result, the search process may be broken into trial steps.

The effectiveness of search procedures is assessed using indices for the reliability with which the operation is performed and a quantity which characterizes the "effort" expended.

The effort expended in searching can be characterized by the time used in searching a given region, the quantity of energy expended, etc.

A search operation may be terminated in an incorrect decision because of interference (e.g., noise). The reliability of the operation can be assessed, for example, by the probabilities of correct and incorrect decision.

At the present time there has been comparatively little work on the synthesis of optimum search systems, i.e., optimum in the sense of minimum expenditure of effort to achieve given reliability indices or in the sense of achieving the best indices for a fixed expenditure of effort (see References 1, 2, 8).

Obviously, a search method employing identical trial steps (sample lengths) may differ significantly from the optimum procedure because of nonuniform a priori distributions, differences in the values of correct and incorrect decisions, etc.

Concurrent with the investigation of general methods for the synthesis of optimal search systems it is expedient to consider specific methods for improving the effectiveness of search procedures. Various methods of controlling the duration of the trial steps (samples) are considered below.

2. In a system admitting a finite number of discrete states, the control of sample duration can be achieved by different methods. We will consider three methods of controlling sample duration, i.e., three procedures for sequential search: a) the use of statistical sequential analysis; b) the use of grouped sequential procedures; c) selection of some elements for further analysis.

In the first method, the information received previously at each trial position (element) is used to form a current likelihood coefficient. The value of this likelihood coefficient for the last interval of time is compared with thresholds, and when either the upper or lower threshold is reached search is transferred to the following element.

The second method differs from this in that the likelihood coefficient is not calculated and compared with thresholds during each discrete interval, but only once for a given group of intervals. After comparison with the thresholds, either a transition occurs to a new element or termination of the current step is delayed for the duration of the following group.

The third method of element selection is based on division of the search (frame) time into stages. During the first stage all elements are

sampled, then a portion of the elements is eliminated and only the residue is tested during the second stage, etc.

It is not difficult to see that the optimum procedure for discrete systems is to control the sample duration using the algorithm of sequential analysis. Indeed, for a discrete system the minimum time (or energy) expenditure in surveying an aggregate of elements, is achieved when the expenditure per element is minimum. But, according to a theorem of Wald-Wolfowitz (Ref. 5), for a given index of reliability, the minimum sample time per element is achieved by using the likelihood coefficient with a double threshold. Thus, the procedure which employs samples of controlled duration based on the algorithm of sequential analysis is optimum in the above sense.

One feature of this class of search techniques is the random nature of the effort expended in searching. Thus, in analyzing these techniques, one wishes to define both the average expenditure of effort as a function of the number of possible decisions (forms of signal), and statistical properties associated with the random character of this effort (for example, the dispersion of the search time or energy).

3. The principal of operation of a search system with controlled sample duration can be illustrated by the example of a system for discretely and automatically tuning a receiver. The functional block diagram of such a system contains a selective tuning element, a double threshold analyzer, and a control system (see Fig. 1). During a trial step, the selective tuning element (filter) remains at a fixed frequency until the comparison procedure is completed. Then the control system, upon command, switches the tuning to a new position (or switches the system from a search to a tracking mode).



We will obtain a relationship which characterizes the effectiveness, in the sense of reducing search time, of procedures employing controlled sample duration.

The mean time to survey  $n$  elements in the absence of a signal is

$$\bar{T}_S = n \bar{\Delta t}_S (D, F). \quad (1)$$

When a signal is present in one of the elements

$$\bar{T}_{CS} = (n - 1) \bar{\Delta t}_S + \bar{\Delta t}_{CS} (D, F) \quad (2)$$

where  $D$  is the required probability of detecting a signal, and  $F$  is the permissible false alarm probability.

In the case of search with a large number of elements ( $\bar{T}_{CS} \approx \bar{T}_S$ ), the effectiveness of the search will be characterized by the quantity:

$$\eta_t = \frac{T_o}{\bar{T}_S} \approx \frac{\Delta t_o}{\bar{\Delta t}_S} \quad (3)$$

where  $\Delta t_o$  is the length of a sample when all samples have the same duration.

The following estimate, obtained from Ref. 4, can be used to calculate the reduction in sample duration achieved by using the double threshold sequential procedure when  $F \approx 1 - D$ :

$$2 < \eta_T < 4$$

$$\eta_T \rightarrow 4 \text{ for } F = (1 - D) \rightarrow 0$$

For  $F \ll (1 - D)$ , as shown in Ref. 3:

$$\eta_T \approx \frac{\ln F}{\ln(1 - D)} \quad (4)$$

The influence of randomness on the search (frame) time can be characterized by the standard deviation of the search time from its mean value.

Since the sample times for the separate elements are independent, the relative magnitude of the standard deviation of search time is given by the relationship:

$$\delta_T = \frac{\sigma_T}{\bar{T}} = \frac{1}{\sqrt{n}} \frac{\sigma_{\Delta t}}{\bar{\Delta t}} \quad (5)$$

where  $\sigma_{\Delta t}$  is the standard deviation of sample length for the sequential procedure (see Refs. 3 and 6); so long as one is interested in a region where  $\sigma_{\Delta t} < 3 \bar{\Delta t}$ , then for  $n > 100$ ,  $\sigma_T < 0.3 \bar{T}$ .

The reduction in search time inherent in the use of sequential search procedures can be realized instead as a reduction in energy expenditure (e.g., by reducing the signal threshold levels).

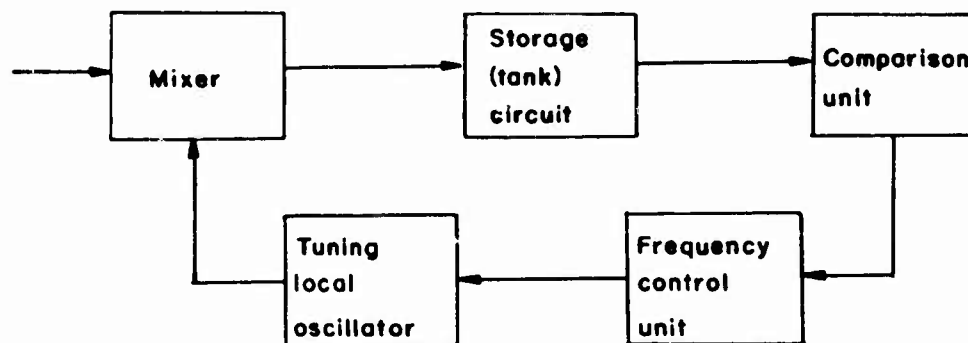


Fig. 1 — Functional schematic of an automatic tuning system

Redistributing the length of sample intervals depending on whether a signal is present or absent, allows one to increase the sample time for elements containing a signal, while maintaining the total search time constant.

We will obtain a relationship defining the mean sample duration under the condition of constant total search time.

Solving the equation for  $\overline{\Delta t}_{CS}^*$ :

$$(n - 1) \overline{\Delta t}_S^* + \overline{\Delta t}_{CS}^* = n \Delta t_0$$

$$\overline{\Delta t}_{CS}^* = \xi \overline{\Delta t}_S^* \quad (6)$$

( $\xi = f(D, F)$  is the coefficient of asymmetry for the sequential procedure) we obtain:

$$\frac{\overline{\Delta t}_{CS}^*}{\Delta t_0} = \frac{\xi n}{(n - 1) + \xi} \quad (7)$$

We define:

$$\zeta = \frac{\Delta t_0}{\overline{\Delta t}_{CS}^*} \quad (8)$$

$\overline{\Delta t}_{CS}$  is the mean sample duration when a signal of threshold intensity is present, i.e., corresponding to the intensity for the case of constant sample lengths. The magnitude  $\zeta = \Delta t_0 / \overline{\Delta t}_{CS}^* > 1$  (for  $F \ll (1 - D)$ ,  $\zeta \approx \frac{1}{D}$ ). From Eqs. (7) and (8), we obtain an expression for the gain in sample time for elements which contain a signal:

$$\eta_t = \frac{\overline{\Delta t_{CS}^*}}{\overline{\Delta t_{CS}}} = \zeta \frac{\xi n}{(n-1) + \xi} \quad (9)$$

$$\eta_t \approx \zeta \xi \text{ for } n \gg 1$$

We will estimate the reduction in signal threshold level for controlled search, resulting from the redistribution of mean sample duration, for both the case of a coherent and an incoherent receiver.

In the case of coherent integration, i.e., with the best possible utilization of signal energy, it follows from a relationship given in Ref. 6 that:

$$q_k^2 = \frac{2 \left[ D \ln \frac{D}{F} + (1-D) \ln \left( \frac{1-D}{1-F} \right) \right]}{\overline{\Delta t_{CS}}} \quad (10)$$

where  $q_k^2 = E/N_0$ , the ratio of signal energy received in an individual sample to the spectral density of noise,  $N_0$ . From (9) and (10) we obtain:

$$\eta_Q = \frac{q_k^2}{(q_k^*)^2} = \eta_t \quad (11)$$

In the case of incoherent integration of weak signals, because of losses resulting from signal suppression, it follows from Ref. 6 that:

$$q_k^4 = \frac{2 \left[ D \ln \frac{D}{F} + (1-D) \ln \left( \frac{1-D}{1-F} \right) \right]}{\overline{\Delta t_{CS}}} \quad (12)$$

$$\eta_Q \approx \sqrt{\eta_t} \quad (13)$$

Considering that for  $F \ll D$ :

$$\xi = \frac{D \ln \frac{D}{F} + (1 - D) \ln \left( \frac{1 - D}{1 - F} \right)}{F \ln \frac{D}{F} + (1 - F) \ln \left( \frac{1 - D}{1 - F} \right)} \approx \frac{D \ln \frac{D}{F}}{\ln(1 - D)} \quad (14)$$

we obtain estimates for the energy-effectiveness of search procedures in the case of non-symmetrical error costs, in the form:

$$\sqrt{\frac{\ln F}{\ln(1 - D)}} \leq \eta_q \leq \frac{\ln F}{\ln(1 - D)} \quad (15)$$

The asymptotic values of the upper and lower bounds for the symmetrical case where  $(1 - D) = F \rightarrow 0$ , according to Eqs. (3), (11), and (13) have the form:

$$2 \leq \eta_q < 4$$

4. Radar search is an important example of the application of sequential detection techniques (Ref. 8).

Assume the antenna beam is step scanned, with the duration of each angular step controlled by a double threshold decision circuit (Fig. 2).

After a decision is made to change the beam position, the storage element is cleared and then the process is repeated for the new beam position.

A peculiarity of the controlled search mode in this case is the presence of many signal coordinates or parameters (e.g., range or doppler cells) at each sample step. In effect, the echo signal has a multi-dimensional form, requiring a multi-channel receiving system.

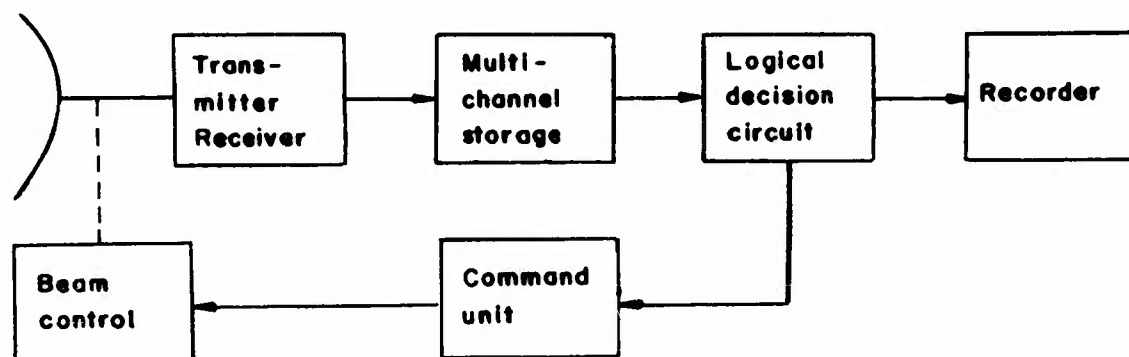


Fig. 2 — Functional block diagram of a system  
with controlled search

We will consider a relationship which characterizes the effectiveness of controlled search in this case.

Assume that the complete search zone contains  $n$  elementary sectors (or beam positions). The mean frame time (i.e., time to search the complete zone) in the absence of a signal is:

$$\bar{T}_S = n \Delta t_S \text{ (m)}$$

When  $s$  targets are present, the mean frame time becomes:

$$\bar{T}_{CS}(s) = (n - s) \bar{\Delta t}_S + \sum_{i=1}^s \bar{\Delta t}_i \quad (16)$$

For  $s \ll n$ ,  $\bar{T}_{CS}(s) \approx \bar{T}_S$ . Thus, when there is a large number of sectors (i.e.,  $n$  is large) in the search zone, the mean search time is determined

by the time required to search in "empty" directions.

Equation (9) gives the saving in search time as compared with the case of uncontrolled search and equal sample durations, assuming equivalent indices of reliability.

For an  $m$ -channel system, Eq. (9) becomes:

$$\eta_t(m) = \eta_t(1) \frac{1}{\vartheta(m)} \quad (17)$$

where:

$$\vartheta(m) = \frac{\psi(m)}{\chi(m)} ; \quad \psi(m) = \frac{\overline{\Delta t}_S(m)}{\overline{\Delta t}_S(1)} ; \quad \chi(m) = \frac{\Delta t_o(m)}{\Delta t_o(1)}$$

The mean duration of the sequential procedure in the multi-channel case has, in accordance with Eq. (9), a linear-logarithmic dependence on the number of channels,  $m$ :

$$\overline{\Delta t}_S(m) = \overline{\Delta t}_S(1) [1 + k \ln m] \quad (18)$$

The dependence of signal threshold level on the number of channels has been considered in a number of works (see e.g., Ref. 10) for the single threshold procedure. From Ref. 10:

$$q_H^2(m) = q_H^2(1) (1 + \gamma \ln m) \quad (19)$$

where  $\gamma$  depends on the signal form (e.g., for a Rayleigh distributed signal,

$$\gamma \approx \frac{1}{\ln \frac{1}{F} - \ln \frac{1}{D}}).$$

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\* Estimating the coefficient  $K$  by a statistical experimental method (Ref. 9) gives:  $k \approx 1$  for  $D \leq 0.5$ ;  $0.5 < k < 1$  for  $D > 0.5$ .

For coherent integration  $\chi(m) = q_K^2(m)/q_K^2(1)$ ; for incoherent integration  $\chi(m) \approx \left[ q_H^2(m)/q_H^2(1) \right]^x$ , where  $1 < x \leq 2$ . Estimates of the energy saving due to the use of sequential detection can be obtained from Eqs. (11) and (13).\*

As an example, Table 1 gives the calculated values of the coefficient  $\eta_1$  for a false alarm probability,  $F$ , of  $10^{-6}$ .

Table 1

m	Coherent Mode		Incoherent Mode	
	D = 0.5	D = 0.9	D = 0.5	D = 0.9
1	19.8	6	4.5	2.46
10	7.2	3	2.95	1.9
100	4.95	2.2	2.67	1.76
1000	4.05	1.86	2.56	1.72

5. The search procedure can be simplified by using the data obtained at the end of a fixed interval (group interval) to control the duration of the samples. The number of data points in a group interval obviously is equal to the ratio  $\Delta_{gp}/\Delta_\tau$ , where  $\Delta_\tau$  is the basic discrete time interval.

Use of this grouping technique leads to discrete changes in the sample duration, since sample duration is a multiple of the group duration.

It can be shown that the mean sample duration achieved with this

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\*The estimates of the effectiveness of controlled search (i.e., sequential detection) obtained from Eqs. (9), (17), and (18) are in good accord with the results presented in Ref. 8.



grouping technique is, in the worst case, no greater than the mean sample duration for sequential analysis plus twice the group duration (Ref. 5).

With high confidence one can use the estimate:

$$\overline{\Delta t}_{rp} \leq \overline{\Delta t} + \Delta_{rp} \quad (20)$$

Using Eq. (20), we obtain the following condition for choice of the group duration:

$$\Delta_{rp} < (0.2 - 0.3) \overline{\Delta t}_S \quad (21)$$

If Eq. (21) is satisfied, one can neglect the "losses" associated with the use of a grouping procedure.

This search procedure, controlling sample length by the use of data groups, can be presented in the form of an equivalent multi-stage process: a) in the first stage all elements are sampled for the duration of a single data group, and using double threshold comparison a subset of elements is selected for further testing; b) during the second and following stages of the process, elements which were not previously eliminated are tested for a group interval  $\Delta_{rp}$ ; c) single threshold comparison is used during the final stage of this process.

Using a group interval equal to the basic discrete time interval (i.e.,  $\Delta_{rp} = \Delta_1$ ), we obtain a multi-stage search process equivalent to the process with continuous control of sample duration.

One can formulate the above condition as a statement of the energy-equivalence of multi-stage search processes and search with continuous control of the sample duration.

6. One of the variants of the multi-stage process is a method based on single threshold selection.

The algorithm of the multi-stage procedure can be stated in the following form: a) in the first step, as a result of equal duration sampling, elements are selected in which the output exceeds a threshold  $A_1$ ; b) in the second step only these selected elements (i.e., those for which  $u_1 > A_1$ ) are tested and a subset which exceeds a threshold  $A_2$  is selected from among these, etc., throughout a number of steps,  $k$ , specified beforehand.

The reliability of this search procedure is characterized by the quantities:  $D$ , the probability of detecting a signal

$$D = \prod_{i=1}^k D_i \quad (22)$$

and  $F$ , the false alarm probability

$$F = \prod_{i=1}^k F_i \quad (23)$$

Because of the randomness of the number of elements sampled during each step, the energy expenditure for each separate operation has a random magnitude

$$Q_k = \sum_{i=1}^k \mathcal{L}_i Q_i(D_i, F_i) \quad (24)$$

where  $\mathcal{L}_i$  is the number of elements tested during the  $i^{\text{th}}$  step (the number

$\mathcal{L}_1$  has a binomial distribution);  $Q_1(D_1, F_1)$  is the ratio of signal energy to noise spectral density.

The mean energy expended in the absence of a signal is given by the relationship

$$\bar{Q}_k = \mathcal{L} \left[ Q_1 + F_1 Q_2 + F_1 F_2 Q_3 + \dots + \prod_{i=1}^{k-1} F_i Q_k \right] \quad (25)$$

For a fixed number of steps, the choice of threshold levels and distribution of energy by steps can be determined from the condition of minimizing the mean energy expended, with the constraint of given indices of reliability.

The reduction of energy expended as compared with the case of single-step search using constant sample duration can be expressed by the coefficient  $\eta_Q = Q_1(D, F) / \bar{Q}_k(D, F)$ , where  $Q_1(D, F)$  is the energy expended in constant sample search.

As an example, we will consider the optimization of energy distribution for the case of a 2-stage search procedure. The problem of choosing an optimum distribution in this case amounts to finding an extremum of Eq. (25), by varying the values of  $F_1$  and  $D_1$ .

a) Assume one is searching for a signal of completely known form in one of  $\mathcal{L}$  possible positions. The expected signal is a sine wave of known amplitude.

The energy expenditure, expressed as a ratio of signal energy to noise spectral density, in accordance with Eq. (25), may be given in the form

$$\bar{Q}_2(D, F) = \mathcal{L} \left[ (\lambda_{D1} + \lambda_{F1})^2 + F_1(\lambda_{D2} + \lambda_{F2})^2 \right] \quad (26)$$

where  $\lambda_\alpha$  is related to the normal distribution

$$\alpha = \frac{1}{\sqrt{2\pi}} \int_{\lambda_\alpha}^{\infty} e^{-x^2/2} dx \quad (27)$$

$$D = D_1 D_2; \quad F = F_1 F_2$$

The optimum values of the parameters  $D_1$  and  $F_1$  are obtained by setting:

$$\frac{\partial \bar{Q}_2}{\partial D_1} = 0, \quad \frac{\partial \bar{Q}_2}{\partial F_1} = 0 \quad (28)$$

The solution of Eq. (28), taking Eq. (27) into account, was obtained graphically. Table 2 gives the calculated values of  $\eta_Q$ , the estimated gain as compared with single-stage equal-sample search, for an optimum choice of the parameters  $D_1$  and  $F_1$ .

Table 2

$D_1$	$F_1$		
	$10^{-4}$	$10^{-6}$	$10^{-8}$
0.5	3.38	4.74	6.36
0.9	2.46	3.58	4.13

b) Next consider searching for a fluctuating signal with Rayleigh amplitude distribution, occurring in one of  $\mathcal{L}$  possible positions, when there are  $m$  receiver channels in each position. As shown in Ref. 10, the relative intensity of threshold signals in this case for optimum processing, can be expressed quite accurately by the equation:

$$Q_2 = \frac{|\ln F| - \ln m}{|\ln D|} - 1 \quad (29)$$

The equation for energy expenditure (26), with the substitution of Eqs. (27) and (29) takes the form

$$Q_2 = \mathcal{L} \left[ \left( \frac{|\ln F_1| - \ln m}{\ln |D_1|} - 1 \right) + F_1 \left( \frac{\ln \left| \frac{F}{F_1} \right| - \ln m}{\left| \ln \frac{D}{D_1} \right|} - 1 \right) \right] \quad (30)$$

The equation for the optimum choice of  $D_1$  and  $F_1$  can be obtained using Eqs. (28) and (30).

The calculated value of the coefficient  $\eta_Q$ , assuming optimum values for  $D_1$  and  $F_1$ , is given in Table 3.

Table 3

$D_1$	$F_1$					
	$10^{-4}$	$10^{-6}$	$10^{-8}$			
	$m = 1$	$m = 1$	$m = 1$	$m = 10$	$m = 100$	$m = 1000$
0.5	2.05	2.54	3.2	2.45	2.0	1.78
0.9	1.78	2.37	2.96	2.36	1.95	1.75

Comparison of the data presented in Tables 2 and 3 shows that the two-step search process is more effective (in comparison to a single stage process) in the case of a signal with constant amplitude than in the case of a fluctuating signal.

Increasing the allowed capacity ( $m$ ), as in the case of double-threshold-controlled sequential detection (see Table 1), leads to a reduction in the difference between equal-sample and controlled modes of search.

### CONCLUSIONS

It was shown that controlled search techniques can improve the efficiency of signal detection. In particular, for a given frame time, controlling the direction of search allows one to reduce the average radiated power.

Allowing a variety of possible signal forms (the capacity permitted in delay time and doppler spread, fluctuations in signal intensity) reduces the gain achieved by using controlled search procedures.

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